

An Introduction to Stochastic Calculus

Fall Semester 2009-2010

Programme

1. **Reviewing the basics of stochastic processes:** What is a stochastic process. Finite dimensional distributions. Kolmogorov's theorem. Sample paths. Basic examples.
2. **The Brownian motion:** Different equivalent definitions. A construction. The quadratic variation process. Sample path properties. Martingale property. Markov and strong Markov property.
3. **The stochastic integral:** Itô's stochastic integral with respect to the Brownian motion. Construction and main properties. The Stratonovich integral. Relation between Itô's and Stratonovich's integrals.
4. **Itô's formula:** A change of variables formula. The role of the quadratic variation.
5. **Some applications of Itô's formula:** Burkholder-Davis-Gundy inequalities. Lévy's characterization of Brownian motion. Itô's representation of square integrable Brownian functionals. Girsanov's theorem.
6. **Tanaka's formula:** Local time of Brownian motion. Tanaka's formula.
7. **Stochastic differential equations:** Some motivating examples. Solving linear equations. A theorem on existence and uniqueness of solution. Dependence on the initial condition. Sample path properties of the solution. Markov property. Euler approximations of SDEs
8. **Time continuous martingales:** A brief introduction to this class of processes and their main properties. Doob's inequalities. Local martingales. The quadratic variation of a L^2 , sample path continuous martingale. Itô calculus with respect to continuous martingales.

References

- P. Baldi: Equazioni differenziali stocastiche e applicazioni. Quaderni dell'Unione Matematica Italiana 28. Pitagora Editrici. Bologna 2000.
- R. Durrett: Stochastic Calculus, a practical introduction. CRC Press, 1996.

J.-F. Le Gall: Mouvement brownien et calcul stochastique. Notes de Cours de DEA 1996-1997. <http://www.dma.ens.fr/legall/>

I. Karatzas, S. Shreve: Brownian Motion and Stochastic Calculus. Springer Verlag, 1991.

B. Øksendal: Stochastic Differential Equations; an Introduction with Applications. Springer Verlag, 1998.

D. Revuz, M. Yor: Continuous martingales and Brownian motion. Springer Verlag, 1999.